Numerieke wiskunde 2, WINM2-08 2011/12 semester II a

Examination, April 2nd, 2012.

Name	**Student number**

Notes:

- You may use one sheet (single side written) with notes from the lectures.
- During the exam it is NOT permitted to consult books, handouts, other notes.
- Numerical/graphic calculators are permitted, programmable calculators are NOT permited.
- Devices with wireless internet connection and/or document readers are NOT permitted.
- Hint: please describe the solution procedures in full details, not only the results.

TEST (to be returned by 17:00)

1. (*Solution of linear systems*) A matrix $A \in \mathbb{R}^{n\times n}$ is strictly diagonally dominant if

$$|a_{ii}| > \sum_{j=1, j\neq i}^{n} |a_{ij}|, \quad i = 1, \ldots, n.$$

   (a) **[pts 6]** Show that if A^T is strictly diagonally dominant, then A has an LU factorization.

   (b) **[pts 4]** Show that if A^T is strictly diagonally dominant, then for the L factor we have $|\ell_{ij}| \leq 1$.

2. (*Least squares approximation to data*) **[pts 8]** After studying a certain type of cancer, a researcher hypothesizes that in the short run the number y of malignant cells in a particular tissue grows exponentially with time t. That is, $y = \alpha_0 e^{\alpha_1 t}$. Determine estimates for the parameters α_0 and α_1 that best fit, in the sense of least squares, the researcher's observed data given in the table below.

t (days)	1	2	3	4	5
y (cells)	16	27	45	74	122

3. **[pts 8]** Determine the QR factors of the matrix

$$A = \begin{pmatrix} 0 & -20 & -14 \\ 3 & 27 & -4 \\ 4 & 11 & -2 \end{pmatrix}$$

using the standard inner product for $\mathbb{R}^n$ and the classical GramSchmidt procedure (*i.e.* not necessarily the modified GramSchmidt procedure).

4. (*Householder transformation*)

   (a) **[pts 5]** Given any two nonzero vectors x and y in $\mathbb{R}^n$, construct a Householder matrix H such that Hx is a scalar multiple of y.

   (b) **[pts 5]** Judge if the Householder matrix at the point (a) is unique or not. Motivate your answer.

5. (*Polynomial of best approximation to functions*) **[pts 8]** Among all polynomials $p_n \in \mathcal{P}_n$ of the form

$$p_n(x) = \sum_{k=0}^{n} a_k x^k, \quad a_k \in \mathbb{R}, \quad a_n \neq 0,$$

find the polynomial of best approximation for the function $f(x) \equiv 0$ on the closed interval $[-1, 1]$.

6. (*Polynomial of least squares approximation to functions*) Because of the difficulty in calculating the minimax approximation polynomial, we often go to an intermediate approximation called the *least squares approximation polynomial.* As notation, we introduce

$$\|g\|_2 = \sqrt{\int_a^b |g(x)|^2 \, dx}, \quad g \in C[a, b].$$

and we look for a polynomial r_n^* that minimizes the expression:

$$E_n(f) = \|f - r_n^*\|_2 \, .$$

   (a) **[pts 6]** Let $f(x) = e^x$, $-1 \leq x \leq 1$. Compute the polynomial $r_1^*(x) = b_0 + b_1 x$ which minimizes the error $\|f - r_1^*\|_2$.

   (b) **[pts 4]** Estimate the final error of the approximation $\|f - r_1^*\|_2$.